- Projectiles:

A projectile is an object that is launched into the air and then moves predominantly under the influence of gravity. The general motion of a projectile is complicated by air resistance, the rotation of Earth, and the variation in direction and magnitude of the gravitational acceleration.

For simplicity, we will neglect these complications. The projectile then has a constant acceleration directed vertically downward with magnitude 'g', and the path of the projectile remains in a vertical

plane determined by the initial velocity.
To describe the projectile's motion, it is convenient to choose this vertical plane as x-y plane, with the x axis horizontal and the y axis vertically upward. With the only acceleration provided by the gravity, we have $a_1 = 0$, and $a_2 = -g$.
Since there is no horizontal acceleration, the horizontal component of velocity remains same. Therefore the motion is a combination of horizontal motion with constant velocity and vertical motion with constant acceleration. If we choose the origin at the initial position of the projectile, following equations describe its motion.

$$v_1 = u_1 \quad ; \quad x = u_1 t$$

$$v_2 = u_2 - gt \quad ; \quad y = u_2 t - gt^2/2 \quad ; \quad v_2^2 = u_2^2 - 2gy$$

If the initial velocity u makes an angle θ with the horizontal axis, the initial velocity components are, $u_1 = u\cos\theta$ and $u_2 = u\sin\theta$.

We can get a lot of information from these equations. For example:
The time to reach the highest point is found by setting $v_2 = 0$ in the equation $v_2 = u_2 - gt$.

$$t = u_2/g = u\sin\theta/g$$

The time T, to return to the launch level, is found by setting $y = 0$ in $y = u_2 t - gt^2/2$.

$$T = 2u_2/g = 2u\sin\theta/g$$

The maximum height H from the launch level, is found by setting $v_2 = 0$ in the equation $v_2^2 = u_2^2 - 2gy$, or by using the time to reach the highest point, in the equation $y = u_2 t - gt^2/2$.

$$H = u_2^2/2g = u^2\sin^2\theta/2g$$

The horizontal range R, till the projectile returns to the launch level, is found by using the corresponding time in the equation $x = u_1t$

$$R = 2u_1u_2/g = u^2\sin 2\theta/g$$

The equation for the shape of the path (called equation of path or trajectory) is obtained from the equation $x = u_1t$ and $y = u_2t - gt^2/2$ by eliminating the variable t.

$$y = x\tan\theta - gx^2/2u^2\cos^2\theta$$

This is the equation of a 'parabola'.

Notice that for a given initial speed, the maximum value of T and H are $2u/g$ and $u^2/2g$, which occur when $\theta = 90°$. And the maximum value of R is u^2/g, which occurs when $\theta = 45°$. Moreover, the R is same for two complementary values of θ, such as 15° and 75°, that is, for angles equally spaced on either side of 45°.

We emphasize that equation for T, H, and R, and even equation of trajectory, are not the fundamental equations of physics. For example, equation of R doesn't give the horizontal range when the projectile returns to a different height. If you think that these specialized results are on an equal footing with more fundamental equations and principles, then you are missing the big picture of a science with a few principles from which all else follows.

Finally note that we are free in choosing the origin and the orientation of a coordinate system. But we try to use such a coordinate system in which the description of the motion is simplest.